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NUMERICAL ANALYSIS OF PILED RAFTS

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SUMMARY

An approximate numerical method for the analysis of piled raft foundations is presented. The raft is modelled as a thin plate and the piles as interacting non-linear springs. Both the raft and the piles are interacting with the soil which is modelled as an elastic layer. Two sources of non-linearity are accounted for: (i) the unilateral contact at the raft–soil interface and (ii) the non-linear load–settlement relationship of the piles. Both theoretical solutions and experimental results are used to verify that, despite the approximations involved, the proposed method of analysis can provide satisfactory solutions in both linear and non-linear range. © 1998 John Wiley & Sons, Ltd.

Key words: foundation; pile; raft; analysis; case history

1. INTRODUCTION

Foundation engineers have long recognized that piles beneath a raft are often needed only to reduce the settlements of the foundation. To move from the traditional capacity-based design to a settlement-based one, methods of analysis capable of taking into proper account the soil–structure interaction within the pile–raft system are needed. The most rigorous analyses of piled raft foundations are provided by the Boundary Element Method (BEM) or the Finite Element Method (FEM). Butterfield and Banerjee,[1] using a BEM approach, performed an extensive numerical study of the rigid raft resting on rigid or compressible piles embedded in an elastic half-space. More recently, Poulos and Davis,[2] Kuwabara[3] and Bilotta *et al.*[4] have applied the BEM to the analysis of piled raft foundations. Also, the finite element method has been used for the analysis of piled rafts; in most of the cases, however, simple axial-symmetric or plane-strain problems have been solved to reduce the huge computational efforts. Ottaviani[5] used a three-dimensional finite element approximation to analyse a very rigid raft resting on compressible piles embedded in an elastic layer.

When large piled foundations need to be analysed, however, the computational problems are such that the rigour of these methods has to be diluted by some approximations. To this aim a number of simplified approaches have been developed in recent years as practical tools for the design of piled foundations. Approximate hand-calculation methods,[6] based on the use of theoretical solutions, and numerical simplified approaches implemented in computer codes[7,8]

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allow the analysis of foundations resting on a large number of piles. Hooper[9] and Randolph[10] review the available methods and their capabilities.

2. ACCURACY OF THE EXISTING BEM SOLUTIONS

The behaviour of a rigid raft resting on compressible piles embedded in an elastic half-space has been investigated by means of BEM by Butterfield and Banerjee[1] and Kuwabara.[3] A comparison of their findings in terms of settlements is made in Figure 1, for various values of the pile spacing, s, slenderness ratio, L/d, and compressibility, K_{ps}; it shows a satisfactory agreement. On the other hand, the dimensionless plot of the load sharing between the piles and the raft versus the spacing of the piles (Figure 2) shows a large and discouraging disagreement. The portion of the total applied load supported by the piles, as obtained by Kuwabara, is significantly higher than that calculated by Butterfield and Banerjee. The foundations analysed by the Authors are not exactly coincident because of small differences in some of the key parameters. In any case, the slightly larger area of the raft connecting the piles of the group and the higher compressibility of the piles in the Kuwabara's analysis should result in a lower portion of the total applied load carried by the piles, while the opposite occurs. The differences between the two sets of results are probably connected to minor differences in the numerical techniques. According to Randolph,[10] for groups of 100 piles and over, the accuracy of available computer programs is probably not better than $\pm$ 20 per cent. While this is probably not a problem for most engineering purposes, given the

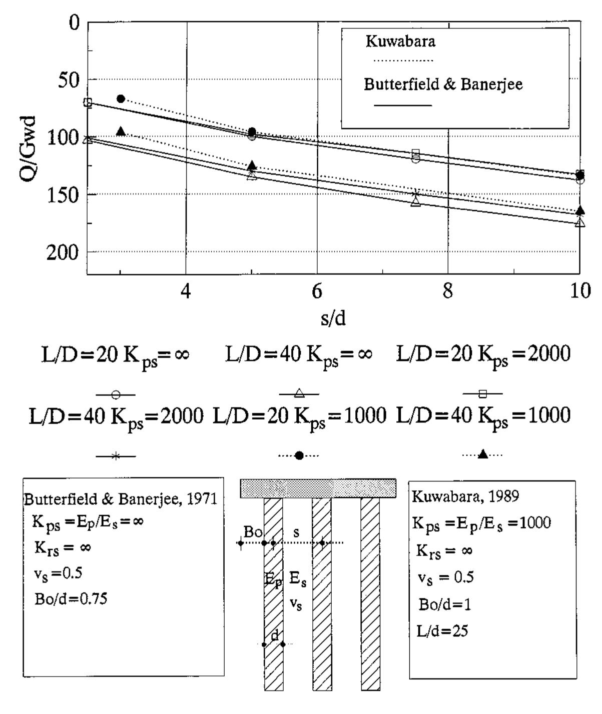

Figure 1. Settlements calculated by Kuwabara[3] and by Butterfield and Banerjee[1]

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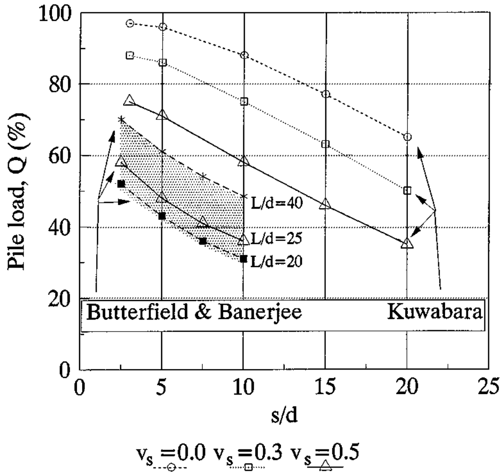

Figure 2. Load shared calculated by Kuwabara[3] and by Butterfield and Banerjee[1]

difficulties and uncertainties involved in estimating the deformation parameters, it still seems to indicate a limit to the extent to which it is fruitful to conduct 'rigorous' analysis.

3. THE COMPUTER PROGRAM NAPRA

3.1. Statement of the problem

In Figure 3 the basic problem considered in this paper is illustrated. A rectangular raft of any flexibility is supported by the soil and by piles in any number and pattern. The raft may be subjected to any combination of vertical distributed or concentrated loading and moment loading. The raft is modelled as a two-dimensional elastic body using the thin plates theory, while the piles and the soil are modelled by means of interacting linear or non-linear springs. It is assumed that the interaction between the raft and the soil (the piles) is purely vertical; accordingly, only the axial stiffness of the springs is required.

The lumped stiffness of the soil springs is deduced by closed form solutions for the settlement of an uniformly loaded rectangular area at the boundary of an homogeneous elastic half-space. The Steinbrenner approximation is used to interface and an iterative procedure sets to zero any tensile force developed between raft and soil.

A stepwise incremental procedure is used to simulate the non-linear load–settlement relationship of the piles.

The method has been implemented in the computer code Non-linear Analysis of Piled RAfts (NAPRA).

3.2. FEM analysis of the raft

The raft is analysed by FEM, adopting a four node rectangular element.[11,12] The element is based on thin plate approximate theory which does not allow for transverse shear strains. This

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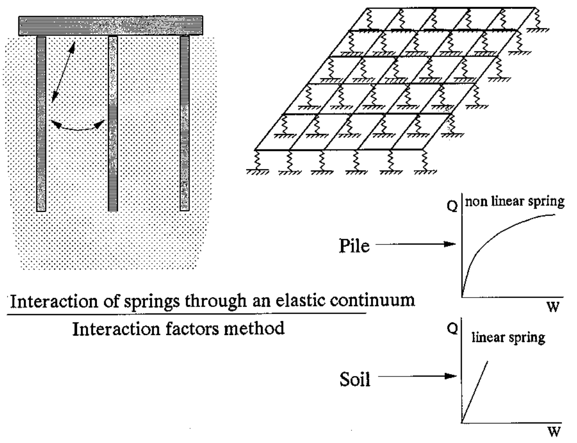

Figure 3. Basic features of the model for a piled raft

rectangular element can also be used to model raft of any geometry adopting a piecewise approximation. The equation of bending for thin plates may be written as follows:

$$D \cdot \nabla^4 w(x, y) = q(x, y) \tag{1}$$

where $w(x, y)$ is the unknown vertical displacement of the raft, $q(x, y)$ is the applied load and D is the bending stiffness. The parameter D is a function of the Young's modulus, E_r, of the thickness, t, and of the Poisson's ratio, ν_r, of the raft as expressed below

$$D = \frac{E_r \cdot t^3}{12 \cdot (1 - \nu_r^2)} \tag{2}$$

In the finite element approach equation (1) is written in terms of a finite number of nodal displacements as follows:

$$[K_r]\{w_r\} = \{q\} \tag{3}$$

where $[K_r]$ and $\{w_r\}$ are, respectively, the stiffness matrix and the vector of the unknown nodal displacements of the raft, while $\{q\}$ represents the vector of the nodal forces or moments acting on the raft.

As the four node rectangular elements used in the analysis have four degrees of freedom at each node, the stiffness matrix of the raft is a square $4n \times 4n$ matrix, where n denotes the number of nodes used to model the raft. If any boundary condition is fixed the stiffness matrix incorporates it.

3.3. *Closed form solutions for soil displacements*

The soil supporting the raft is assumed to be an elastic continuum. The Boussinesq's solution for a point load and the closed form solution for a rectangular uniformly loaded area at the surface of an elastic half-space[13] are used to calculate the soil displacements produced by the

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contact pressure developed at the interface between the raft and the soil. The point load solution is used to calculate the displacement occurring in the point J, due to the contact pressure developed at the interface of the element i, whose resultant is lumped in the central point I. To calculate the displacement occurring at a point I, due to the contact pressure acting on the element whose centre is just the point I, the rectangular uniformly loaded area solution is used.

The layered continuum is solved by means of the Steinbrenner approximation. Basically, it works on the simple assumption that the stress distribution within an elastic layer is identical with the Boussinesq's distribution for an homogeneous half-space. The accuracy of the Steinbrenner approximation has been checked by different authors[14,15] who concluded that it is generally acceptable for engineering purposes.

The above solutions allow to build up the flexibility matrix $[F_s]$ of the soil. This matrix relates the vector of the unknown nodal soil displacements $\{w_s\}$ to the nodal vertical interaction forces $\{r_{rs}\}$ as follows:

$$[F_s]\{r_{rs}\} = \{w_s\} \tag{4}$$

3.4. Piles as non-linear interacting springs

A considerable simplification for the analysis of piled raft foundations is to consider each pile as a single unit. An important feature of the proposed model is that any observed non-linear behaviour of the pile–soil system may be matched by a suitable selection of the empirical load–settlement curve. Caputo and Viggiani[16] claim that the non-linearity of pile–soil interaction overwhelms all the other factors causing an overall non-linear response of a piled foundation.

To model the non-linear behaviour of the single pile the analytical expression of the Chin's hyperbola[17] is used in the present method, since most of the available settlement records are closely fitted by this curve.

When applying the procedure to a real case, a model of the subsoil has to be developed. The data obtained from site and laboratory investigation are adapted to a scheme of horizontal layers. The elastic properties of the soil and the failure load of the pile are backfigured from the results of pile load tests, if they are available; otherwise they are evaluated in the usual way from available data.

In the present analysis based on the method of interaction factors, it is assumed that all factors $\alpha_{ij}(i = j)$ are constant, irrespective of the load level, while the pile–pile interaction factors located on the principal diagonal, α_{ii}, vary according to the following expression:

$$\alpha_{ii} = \frac{1}{1 - (Q_i/Q_{i,\,\mathrm{lim}})} \tag{5}$$

where Q_i is the load acting on the pile i and $Q_{i,\,\mathrm{lim}}$ represents the bearing capacity of the same pile.

The interaction factor method is used to model pile to pile interaction and a preliminary BEM analysis, via a computer code, allows to calculate the interaction factors between a couple of piles at various spacings.[18,19] The interaction factor is defined as follows:

$$\alpha_{\mathrm{pp}}(s) = \frac{w_2(s)}{w_1} \tag{6}$$

where w_i represents the elastic settlement of the pile i, and the load free pile 2 is located at a spacing, s, away from the loaded pile 1.

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Relying upon a large number of numerical results[19] the following continuous curve is used to fit the relationship between the interaction factors

$$\alpha_{pp}(s) = \frac{1}{1 + A\cdot(s/d)^B} + C\,\mathrm{Log}\left(\frac{s}{d} + 10\right) \tag{7}$$

where d is the diameter of the piles.

Details on the fitting procedure and typical values of the parameters A, B, C are reported in the appendix. The two interacting piles may be different both in length and in diameter. Following Randolph and Wroth[20] original suggestion, it is assumed that interaction does not occur for piles whose spacing is larger than a limiting value r_m.

This parameter is defined according to Randolph and Wroth[20] as

$$r_m = \left[0{\cdot}25 + 2{\cdot}5\rho(1 - \nu_s) - 0{\cdot}25)\frac{G_L}{G_B}\right]L \tag{8}$$

where ρ is a parameter which reflects the vertical homogeneity of the soil, varying between 0·5 and 1; G_L and G_B are the values of the shear modulus at depth L (L = pile length) and below the pile base.

When the spacing between two piles is greater than r_m, then the interaction factor α_{pp} is assumed to be zero.

3.5. Interaction between piles and raft elements

Between an axially loaded pile beneath a raft and the raft elements an interaction is developed through the elastic continuum. A sketch of the problem is shown in Figure 4. A BEM procedure has been implemented to calculate the pile–soil interaction factors defined according to equation (6) where the term $w_2(s)$ represents the displacement at the centre of the raft element located at a distance s from the loaded pile. The values calculated at various spacings, s, are again fitted with the continuous curve reported in equation (7), and another set of the three parameters A, B, C is obtained. Other approximate methods[8,21] assume that these factors are equal to the pile–pile

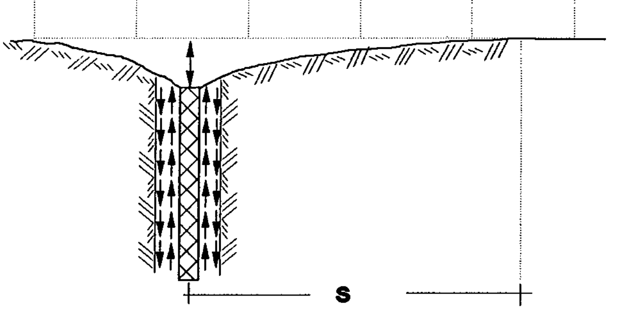

Figure 4. Pile–soil interaction

interaction factors. In the experience of the author this is sometimes a very rough approximation which can significantly affect the calculated load sharing. In the present method the reciprocal theorem is used to maintain that the soil–pile interaction factor is equal to the pile–soil interaction factor.

Furthermore, it is assumed that no interaction occurs when the distance is beyond a limiting value r_m. This parameter is defined again by equation (8).

3.6. *Procedure to solve the model*

The FEM approach to build up the stiffness matrix of the raft has been briefly described in Section 3.2.

The assumption that the mutual interaction between the raft and the pile–soil system is purely vertical allows to reduce the size of the stiffness matrix to the vertical degrees of freedom only by means of a partial backward substitution (see Appendix III). Then for the raft subjected to the pile–soil reaction $\{r_{sr}\}$ equation (3) may be written as follows:

$$[K_r]\{w_r\} = \{q\} + \{r_{sr}\} \tag{9}$$

where the same symbols of equation (3) have been used, even if both the stiffness matrix and the external load vector have been reduced to the vertical degrees of freedom only.

The stiffness matrix $[K_{sp}]$ of the pile–soil system is obtained by inversion of the flexibility matrix $[F_{sp}]$. If the pile–soil is subjected to the raft nodal reaction $\{r_{rs}\}$ then

$$[K_{sp}]\{w_{sp}\} = \{r_{rs}\} \tag{10}$$

The compatibility of the displacements of the raft and the pile–soil system is expressed by the following relationship:

$$\{w_r\} = \{w_{sp}\} = \{w\} \tag{11}$$

and the addition of equations (9) and (10) yields

$$[K]\{w\} = \{q\} \tag{12}$$

where $[K] = [K_r] + [K_{sp}]$.

The linear system (12) is then solved for the unknown displacements.

The main source of non-linearity accounted for is the non-linear load–settlement relationship used for the piles. The stepwise procedure implemented in the program subdivides the total load to be applied into a number of increments, and the diagonal terms of the pile–soil flexibility matrix are updated at each step, according to equation (5).

A computation of the nodal reactions vector $\{r_{rs}\}$ is made at each step, to check for tensile forces between raft and soil and an iterative procedure is used to make them equal to zero. Basically, this procedure releases the compatibility of displacements between the raft and the pile–soil system in the node where tensile forces were detected, although the overall equilibrium is saved by means of a partial backward substitution. An iterative procedure is needed since after the first run some additional tensile forces may arise in different nodes.

The output of the code is represented by the distribution of the nodal displacements of the raft and the pile–soil system, the load sharing among the piles in the group and the soil, and the bending moments in the raft, for each load increment.

A full flow chart for the method is reported in the appendix.

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4. CHECK OF THE PROCEDURE AGAINST KNOWN SOLUTIONS

4.1. Raft on elastic half-space

In order to assess the accuracy of the procedure in the case of a raft simply supported by an elastic half-space without any pile, some well-known benchmarks are used.

Wardle and Fraser[22] studied the bending of a rectangular raft of finite stiffness supported by an elastic half-space, using a refined finite element analysis for the raft coupled with a closed form solution for soil displacements beneath the nodes of the raft mesh. In Figure 5 the results obtained by means of NAPRA for a raft resting on an elastic halfspace are compared with those by Wardle and Fraser. The comparison is made in terms of a dimensionless settlement index against raft–soil stiffness ratio as defined by the authors:

$$K_{rs}(\text{W\&F}) = \frac{4E_r t_r^3 (1 - \nu_s^2)}{3E_s B_r^3 (1 - \nu_r^2)} \tag{13}$$

A general agreement is found. Furthermore, analytical solutions are available for the settlement of an infinitely stiff raft resting on an elastic half-space. For instance, Gorbunov-Possadov and Serebrjanyi[23] give a dimensionless settlement index of 0·88, which compares well with the numerical solutions in Figure 5.

4.2. Piled raft

In the linear range, a check of the proposed procedure is possible against available results obtained by more 'rigorous' procedures, involving very high computational resources. The uncertainties of such solutions, as reported in Section 2, are however to be kept in mind.

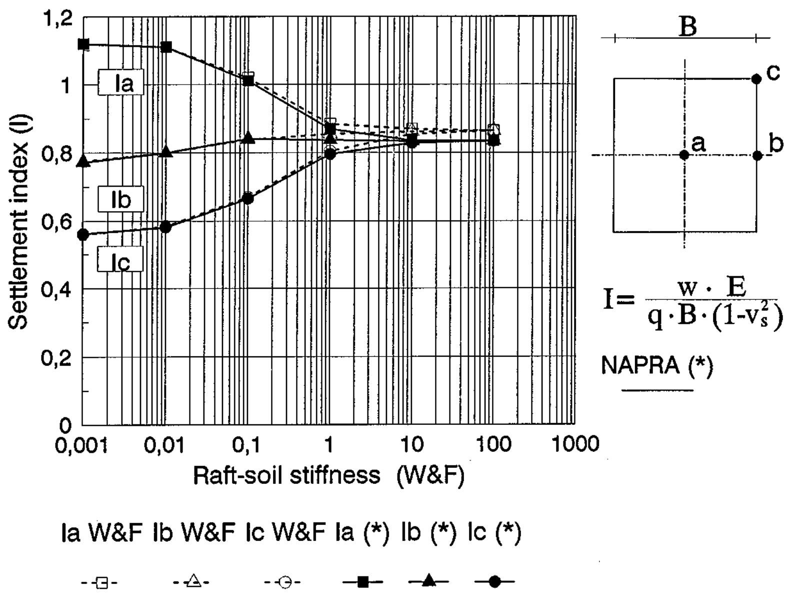

Figure 5. Comparison between solutions for an unpiled raft

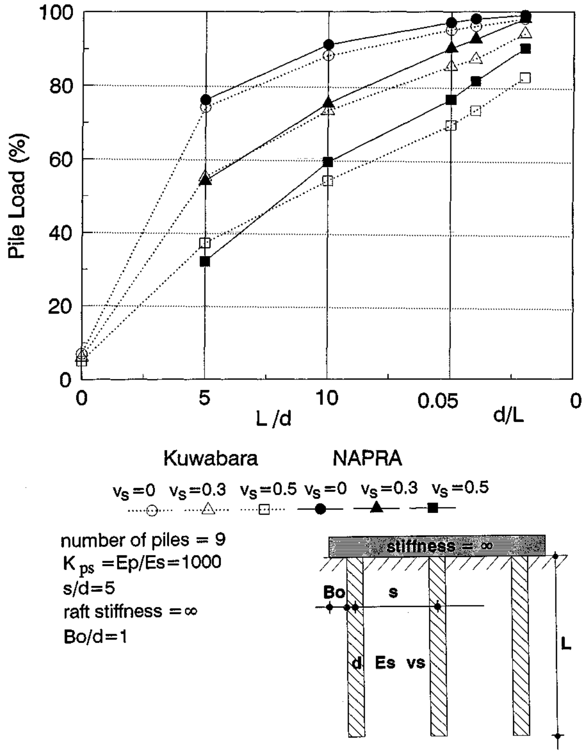

Figure 6. Comparison between solutions for a piled raft

Kuwabara[3] analysed the behaviour of a rigid piled raft on an elastic half-space by a full BEM solution; since the results presented involved large computational resources, the largest foundation analysed was an 8×8 pile group.

Figure 6 shows the load sharing within a piled raft foundation sketched in the same figure and the effects of both the difference between undrained and drained loading, via the Poisson's ratio of the soil, and the slenderness ratio, L/d, of the piles. In the same plot the load sharing calculated with the present method is reported. The general trend of the results is very similar, even if the present method seems to slightly overestimate the percentage of the total load taken by the piles at large values of the slenderness ratio, while the opposite occurs for the lowest values.

5. CHECK OF THE PROCEDURE AGAINST MEASURED BEHAVIOUR

A number of case histories of piled foundations behaviour, monitored in full scale or in centrifuge tests, have been published in recent years. Most of them deal with settlement measurements and only a few show measurements of the mutual interaction among raft, piles and soil. Furthermore, the lack of some essential information makes some of them not suitable for backanalysis. In order to verify the accuracy of the proposed procedure the results obtained by two research project based on centrifuge tests are used in the following two sections.

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Table I. Model boundary conditions[24]

Test	Pile			Raft	
No.	Number	Length (mm)	Diameter (mm)	Breadth (mm)	Thickness (mm)
1	8	135	16·7	180	15
2	12	135	16·7	180	15
3	16	135	16·7	180	15
4	8	135	16·7	180	15
5	8	225	16·7	180	15
6	8	315	16·7	180	15
7	8	225	6·7	180	15
8	8	225	11·7	180	15
9	8	225	16·7	180	15

5.1. *Thaher and Jessberger*[24,25]

Thaher and Jessberger[24,25] tried to figure out the influence of the number of piles, of the pile length, and of the pile diameter on the performance of a square piled raft. The raft and the piles were modelled in the centrifuge using aluminium alloy to simulate at prototype scale the concrete stiffness. Fine sand was glued to the surface of the piles and the raft being in contact with the soil, in order to simulate the roughness of a concrete surface. The tests were carried out using a commercially available kaolin clay and a mean undrained shear strength of 200 kPa was obtained after consolidation. The dimensions of the model used in the tests are summarized in Table I.

A maximum load of 19·44 kN was applied during each test and the observation was continued until the consolidation stage was concluded.

The results obtained from this centrifuge tests were used also by Poulos[8] to verify the accuracy of a proposed numerical method. He suggested to use a limiting shaft friction of 80 kPa, to be compared to the undrained shear strength of 200 kPa. As a matter of fact, the maximum pile head load measured during the test No. 4 by the Authors is about 1·7 kN. Even accounting for some base load, this correspond to a limiting shaft friction not less than 180 kPa.

The Young's modulus of the soil is obtained by fitting the measured final settlement of the unpiled raft with the numerical solution provided by the program NAPRA. In such way a Young's modulus $E_s = 9{\cdot}7$ MPa is obtained and it is retained in all the subsequent analyses.

Figure 7 shows the effect of the number of piles on the performance of the piled raft foundation and compares the predictions by NAPRA and those by Poulos using Garp code with the measured behaviour. It must be noted that

(a) Poulos prediction is linearly elastic in spite of the fact that the load supported by the piles implies a unit shaft friction much higher than the limiting value of 80 kPa assumed by the author;
(b) the analysis by NAPRA is non-linear; a limiting shaft friction of 180 kPa and a Young's modulus $E_s = 9{\cdot}7$ MPa have been assumed;

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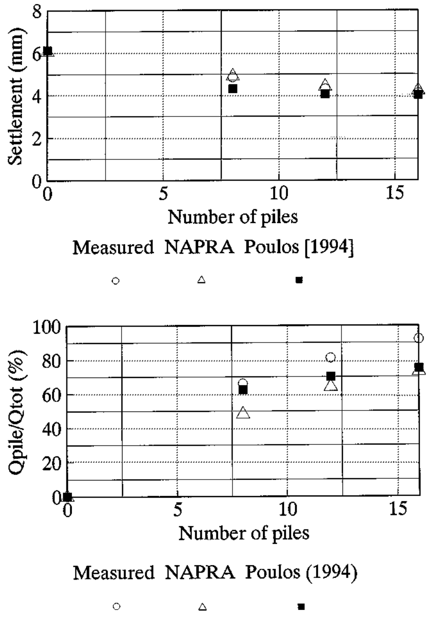

Figure 7. Settlements and load sharing by centrifuge tests and calculation results

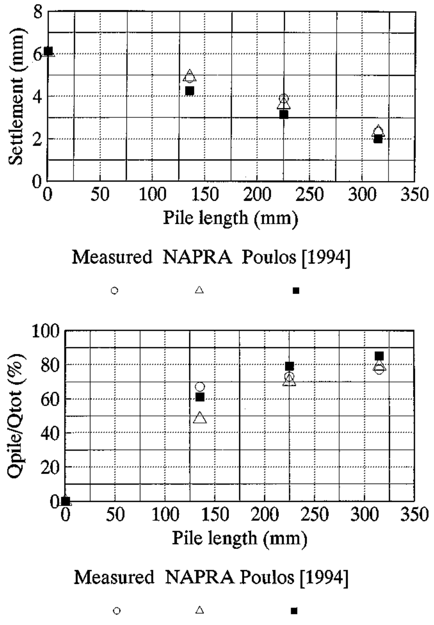

Figure 8. Settlements and load sharing by centrifuge test and calculation results

(c) the average load on the single pile calculated by NAPRA corresponds to an average safety ranging between 1·6 and 2·0.

The settlement of the foundation predicted by NAPRA is very close to the measured one, while the agreement is not so good for the load sharing, with the difference ranging between 16 and 20 per cent.

In Figure 8 the effect of the pile length is shown and again comparisons are made between the measured behaviour and the two prediction methods. As can be seen, the results of the prediction by NAPRA are practically coincident with the measured ones for all but the case with piles 135 mm in length.

Finally, Figure 9 shows the effects of the diameter of the pile. In this case the prediction by Poulos' method is a non-linear one obtained with the above reported 80 kPa ultimate shaft friction.

The behaviour predicted by NAPRA is very close to the measured one both for the settlements and the load sharing while the average safety factor on the single pile ranges between 1·3 and 1·6.

5.2. *Horikoshi*[26]

Horikoshi has performed centrifuge tests for both unpiled and piled circular rafts; the raft is very flexible and supported by piles in various configurations. In the present paper only the case with nine piles located close to the centre of the circular raft will be used to make a comparison.

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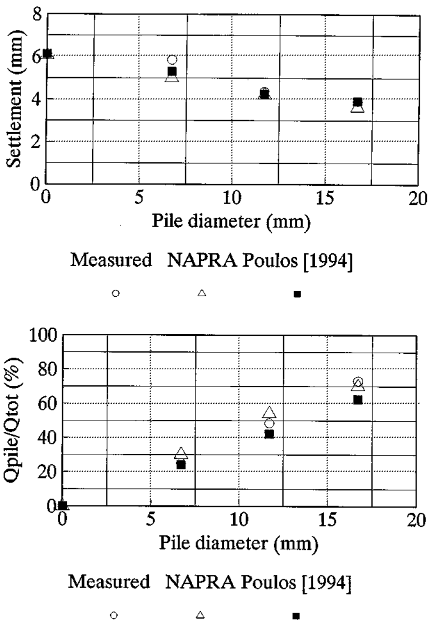

Figure 9. Settlements and load sharing by centrifuge test and calculation results

Table II. Model boundary conditions[26]

Test	Pile		Raft	
No. 1	Number	9	Radius (m)	7·0
	Length (m)	15	Thickness (m)	0·045
	Diameter (m)	0·315	E_r (MPa)	40000
	E_p (MPa)	40000	ν_r	0·16
	ν_p	0·16		

Among the test performed by Horikoshi there are some concerning single capped and uncapped piles, which could be very useful to the present method, since they allow an experimental assessment of both the initial tangent stiffness of the piles and their failure load, which could be used to define the hyperbolic load–settlement relationship for the single pile. Unfortunately, the author states that the initial tangent stiffness deduced by loading tests is not reliable for various reasons. Taking into account the author statement a calculation of the initial tangent stiffness has been performed relying upon the elastic parameters deduced by the soil investigations. A shear modulus $G = 13{\cdot}1$ MPa and a Poisson's ratio $\nu = 0{\cdot}4$ have been used in all the subsequent calculations, while the average experimental failure load $Q_{lim} = 0{\cdot}66$ MN (in the prototype scale) has been retained. In Table II the relevant properties of the model foundation (in prototype scale) are reported.

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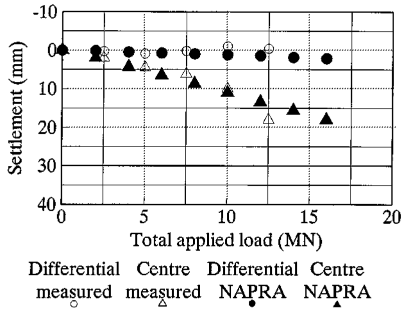

Figure 10. Differential and absolute settlements by centrifuge tests and calculation results

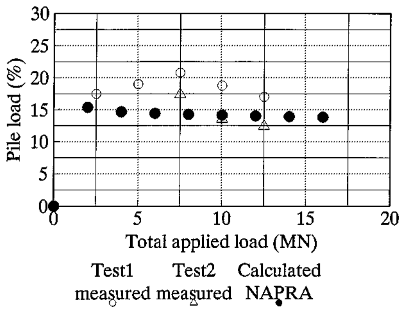

Figure 11. Load sharing by centrifuge tests and calculation results

A finite element piecewise approximation has been used to model a quarter of the symmetrical circular foundation with a total of 196 plate bending elements. The set of results obtained by NAPRA are compared with the measured behaviour in Figures 10–12.

Figure 10 shows the comparison in terms of maximum absolute and differential settlement of the foundation. The agreement between the predicted and measured behaviour is very satisfactory. In Figures 11 and 12 the calculated pile load is compared to the measured one. Figure 11 refers to the total pile load in percentage on the total applied load and the two sets of experimental results refer to two different tests performed on the same model foundation. Test 1 shows that the pile group has initially a low stiffness which grows up with increasing the total applied load. This may be explained in terms of a poor contact between raft and pile caps or of the influence of previous load cycles. The pile load calculation by NAPRA, however, compares well with the average measured behaviour. In Figure 12 the load calculated for some piles, is compared to the measured one in test 2. A general and satisfactory agreement may be again found.

6. CONCLUDING REMARKS

A relatively simple and fast numerical procedure for the analysis of piled-raft foundations has been presented in this paper. The raft of any geometry and stiffness is modelled as a thin plate and solved via FEM. Both force and moment loading can be applied to the foundation.

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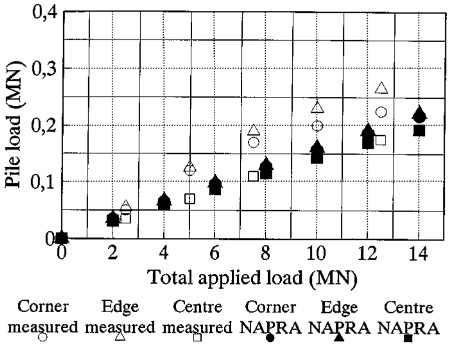

Figure 12. Load sharing among piles in the group and calculation results

The piles and the soil are represented as linear or non-linear interacting springs using the superposition factors and a complete description of the non-linear load–settlement relationship for the single pile is allowed. No-tension contacts between raft and soil are modelled and layered-soil profiles can be solved through Steinbrenner's approximation.

The numerical procedure is implemented via the computer program NAPRA.

It has been shown that theoretical benchmarks are far to provide a totally reliable mean to check the accuracy of a simplified numerical method, even in the linear range. Furthermore, in the non-linear range theoretical benchmarks are missing, and only comparisons with measured behaviour are possible.

The comparisons carried out indicate that the computer program NAPRA may provide satisfactory solutions both in linear and non-linear range. It is important to recognize that when dealing with piled raft designed with piles as settlements reducers, the pile load may be very close to the ultimate bearing capacity and, in any case, well within the non-linear range of the pile–soil interaction response. For this reason allowance was made for a complete but simple description of a non-linear load–settlement relationship for the piles.

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APPENDIX I

Flow-chart of the program—typical values of the parameters A–B–C in equation (7)—partial backward substitution:

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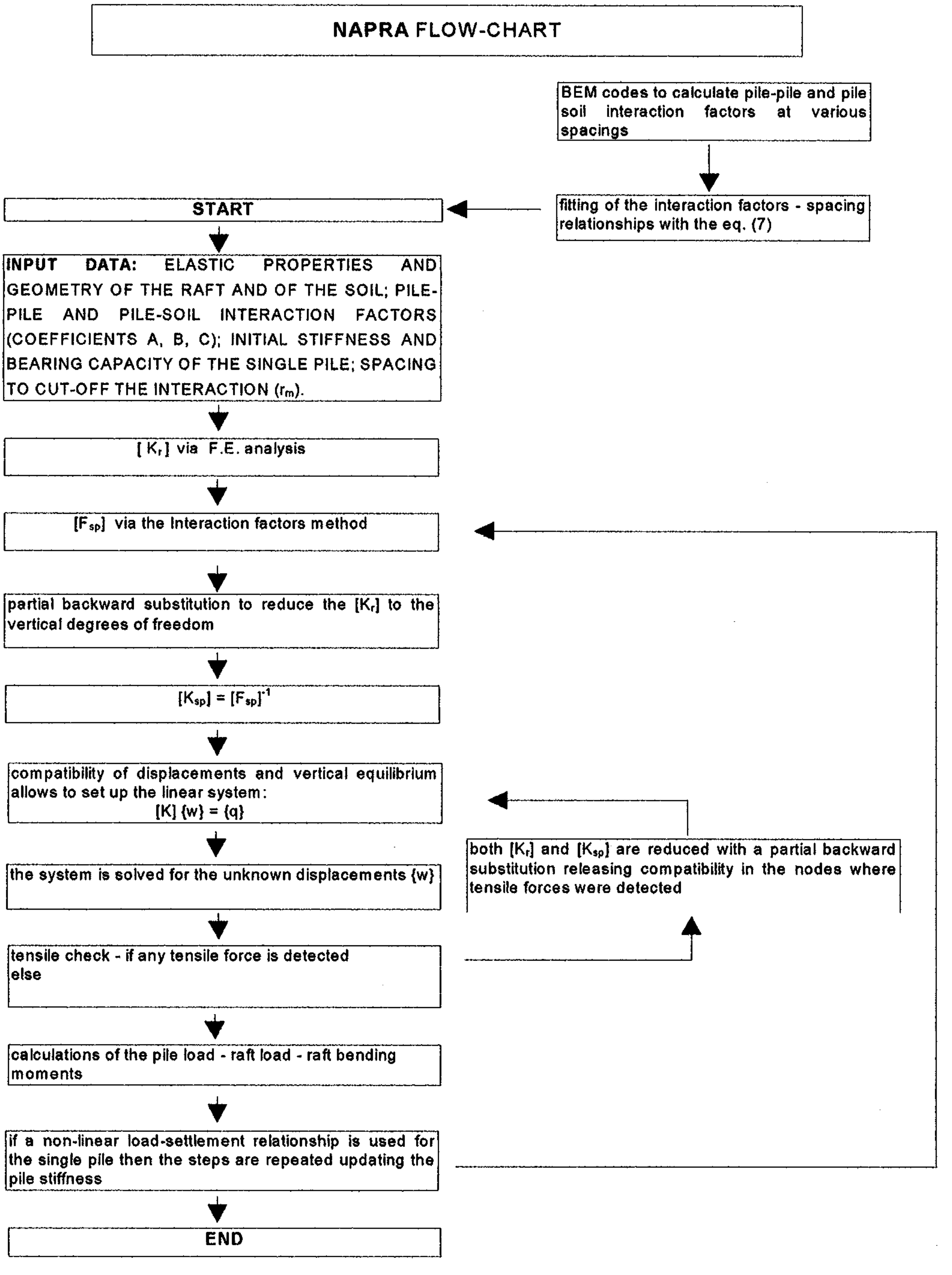
NAPRA FLOW-CHART
BEM codes to calculate pile-pile and pile soil interaction factors at various spacings
fitting of the interaction factors - spacing relationships with the eq. (7)
START
INPUT DATA: ELASTIC PROPERTIES AND GEOMETRY OF THE RAFT AND OF THE SOIL; PILE-PILE AND PILE-SOIL INTERACTION FACTORS (COEFFICIENTS A, B, C); INITIAL STIFFNESS AND BEARING CAPACITY OF THE SINGLE PILE; SPACING TO CUT-OFF THE INTERACTION (r_m).
[K_r] via F.E. analysis
[F_{sp}] via the Interaction factors method
partial backward substitution to reduce the [K_r] to the vertical degrees of freedom
[K_{sp}] = [F_{sp}]$^{-1}$
compatibility of displacements and vertical equilibrium allows to set up the linear system:
[K] {w} = {q}
the system is solved for the unknown displacements {w}
both [K_r] and [K_{sp}] are reduced with a partial backward substitution releasing compatibility in the nodes where tensile forces were detected
tensile check - if any tensile force is detected
else
calculations of the pile load - raft load - raft bending moments
if a non-linear load-settlement relationship is used for the single pile then the steps are repeated updating the pile stiffness
END

APPENDIX II

A BEM code is used to produce interaction factors for a couple of piles located at various of spacings (usually between 3d–60d) or between pile and soil. A commercially available code named 'TABLE CURVE' distributed by JANDEL is used to produce the best fitting of equation (7) to the numerical values of the interaction factors. Typical values of the parameters *A*, *B*, *C* are provided in the table below with reference to the Horikoshi case:

	A	*B*	*C*
Pile–soil interaction factors	0·2546	0·9266	– 0·0477
Pile–pile interaction factors	0·2795	0·8991	– 0·0474

APPENDIX III

Details about partial backward substitution:

Denoted with v the vertical degrees of freedom and with φ the degrees of freedom associated with rotations around the x-axis, y-axis and the twisting around the z-axis equation (3) may be written

$$\begin{bmatrix} K_{vv} & K_{v\varphi} \\ K_{\varphi v} & K_{\varphi\varphi} \end{bmatrix} \begin{Bmatrix} w_v \\ w_\varphi \end{Bmatrix} = \begin{Bmatrix} q_v \\ q_\varphi \end{Bmatrix} \tag{14}$$

which is equivalent to write

$$\begin{cases} [K_{vv}]\{w_v\} + [K_{v\varphi}]\{w_\varphi\} = \{q_v\} \\ [K_{\varphi v}]\{w_v\} + [K_{\varphi\varphi}]\{w_\varphi\} = \{q_\varphi\} \end{cases} \tag{15}$$

From the second of equations (15) the vector $\{w_\varphi\}$ can be found and the substitution in the first of (15) yields

$$[K_{vv}]\{w_v\} - [K_{v\varphi}][K^{-1}_{\varphi\varphi}][K_{\varphi v}]\{w_v\} = \{q_v\} - [K_{v\varphi}][K^{-1}_{\varphi\varphi}]\{q_\varphi\} \tag{16}$$

Hence in equation (9):

$$\begin{aligned} [K_r] &= [K_w] - [K_{v\varphi}][K^{-1}_{\varphi\varphi}][K_{\varphi v}] \\ \{q\} &= \{q_v\} - [K_{v\varphi}][K^{-1}_{\varphi\varphi}][q_\varphi] \end{aligned} \tag{17}$$

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